

Digital adaptive optics and imaging through deep turbulence

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ABSTRACT

In this paper we study the performance of an atmospheric imaging technique we referred to as digital adaptive optics. The technique consists in two major steps: (1) an optical sensor provides simultaneous measurements of the optical field wavefront phase and intensity distributions (complex field) in the system pupil, and (2) a digital processing approach is used to synthesize a compensated image from the complex field measurement. A numerical analysis of the system performance is provided for an anisoplanatic imaging scenario.

Keywords: atmospheric imaging, adaptive optics, wavefront sensing, image enhancement

1. INTRODUCTION

1.1 Conventional adaptive optics

Regardless of the application of interest, conventional adaptive optics (AO) systems typically perform two tasks: (1) they sense the wavefront aberrations resulting from wave propagation through the random media (e.g. the atmosphere), and (2) they compensate these aberrations using a phase conjugation approach. The components required to perform these tasks typically consist of a wavefront sensor (WFS) such as the widely-used Shack-Hartmann WFS, a wavefront corrector (WFC) – typically a deformable or segmented mirror – and a control device that computes the actuator commands sent to the WFC from the WFS data. This compensation process must be performed at speeds that match or exceed the rate of evolution of the random media. As a result of this requirement conventional adaptive optics systems are usually complex and often costly.

1.2 Limitations of conventional adaptive optics

Performance of AO systems is limited by a number of factors among which wavefront correctors performance have a strong impact. First, WFC's have a limited number of degrees-of-freedom. For example, the number of control channels of a deformable mirror seldom exceeds a few tens across its aperture. This limitation affects the spatial scale of the wavefront features the WFC can compensate and prevents the system from mitigating high-order aberrations (i.e. aberrations with small spatial features). This constrain is especially critical for optical systems with aperture diameter $D \gg r_0$, where r_0 is the Fried parameter [1]. Another restrictive feature of WFC's is the limited amplitude of the wavefront phase they can compensate. This limitation prevents in parts conventional AO systems to be effective under strong (deep) turbulence conditions, which are typical for optical systems operating over long and/or near-horizontal (slant) atmospheric propagation paths. Finally, the limited temporal response of WFC's may prevent them from providing compensation at sufficiently high rates.

Although technological developments have been providing WFC's with higher spatial resolution, increased dynamical range and bandwidth, anisoplanatism [2] remains a fundamental limitation for adaptive optics compensation, especially in deep turbulence propagation scenarios.

In the remainder of this paper we describe an alternative approach to conventional adaptive optics – referred to as digital adaptive optics (DAO) – which alleviates the need for physical WFC devices and their corresponding real-time control hardware, and relieves the system from some the limitations associated to them. In section 2 we present the approach used in DAO systems and discuss their limitations. The DAO technique is then applied to an anisoplanatic imaging scenario and results of numerical analysis are presented in section 3. Finally section 4 draws conclusions.

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2. DIGITAL ADAPTIVE OPTICS

2.1 General approach

The notional schematic in Figure 1 shows the sequential steps required for obtaining a compensated image using the digital adaptive optics approach. Two major steps of the process are as follow:

- **Step 1:** Optical field measurement

The front-end of the DAO system consists of an optical reducer and an optical sensor referred to as complex field sensor (CFS). The CFS provides simultaneous measurements of the optical field wavefront phase and intensity distributions in its pupil, $\varphi(\mathbf{r})$ and $I(\mathbf{r})$ respectively. This sensor is referred to as a complex field sensor since the complex amplitude of optical field $A(\mathbf{r})$ can be represented in the form $A(\mathbf{r}) = |A(\mathbf{r})| \exp\{j\varphi(\mathbf{r})\}$, where $|A(\mathbf{r})| = I^{1/2}(\mathbf{r})$, and both phase $\varphi(\mathbf{r})$ and amplitude $|A(\mathbf{r})|$ functions can be obtained from the sensor measurements. The reducer is used for re-imaging of the DAO system pupil onto the CFS pupil so that $A(\mathbf{r}) \approx A_{in}(M\mathbf{r})$ where $A_{in}(\mathbf{r})$ denotes the complex amplitude of the optical field entering the DAO system. The term M is a scaling factor associated with the beam reducer and $\mathbf{r} = \{x, y\}$ designates a coordinate vector in the system pupil plane. To simplify notation, we assume $M = 1$ and $A(\mathbf{r}) \approx A_{in}(\mathbf{r})$. Section 2.2 provides details about complex field sensing techniques for DAO systems.

As a result of propagation through atmospheric turbulence the wavefront phase $\varphi_{in}(\mathbf{r})$ received at the DAO system's pupil (and measured by the CFS sensor) can be separated into two components:

$$\varphi_{in}(\mathbf{r}) = \varphi_{obj}(\mathbf{r}) + \varphi_{turb}(\mathbf{r}), \quad (1)$$

where $\varphi_{obj}(\mathbf{r})$ is the phase component related to the object of interest (scene) and $\varphi_{turb}(\mathbf{r})$ is the turbulence-induced phase term which needs to be compensated. The second step of the DAO process aims to (1) compensate phase aberrations $\varphi_{turb}(\mathbf{r})$ which degrade the quality of the images produced by the system and (2) preserve phase $\varphi_{obj}(\mathbf{r})$ which is used to synthesize a compensated image.

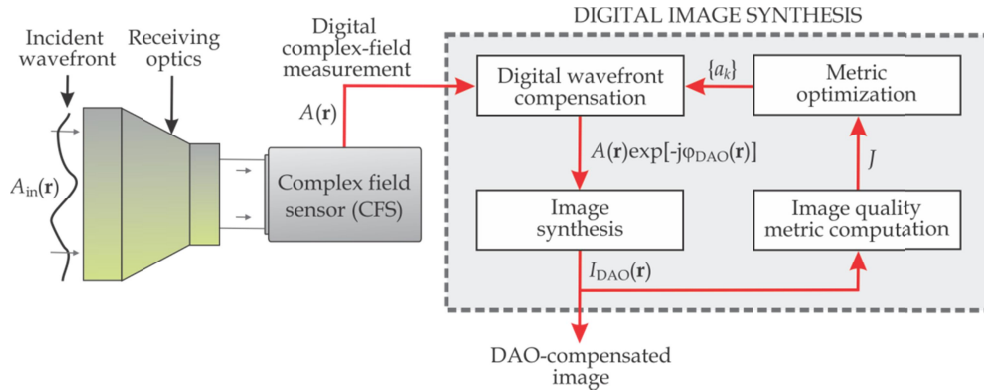


Figure 1. Notional schematic of a digital adaptive optics system.

- **Step 2:** Digital image synthesis

In this second step of the DAO technique, a digital processing technique is used to synthesize a compensated image from the complex field measurement $A(\mathbf{r})$ provided by the CFS (step 1). As illustrated in Fig. 1 this step includes compensation of the measured complex field $A(\mathbf{r})$ using phase function $\varphi_{DAO}(\mathbf{r})$ and results in compensated field $A_{DAO}(\mathbf{r}) = A(\mathbf{r}) \exp\{-j\varphi_{DAO}(\mathbf{r})\}$. This phase conjugation step using numerical phase function $\varphi_{DAO}(\mathbf{r})$ can be regarded as the digital equivalent of phase conjugation using a wavefront corrector such as a deformable mirror or a spatial light modulator which are used in conventional AO systems. We refer to this step as digital wavefront compensation (DWFC).

The compensated complex field $A_{DAO}(\mathbf{r})$ is then used to synthesize image $I_{DAO}(\mathbf{r})$. Image quality of $I_{DAO}(\mathbf{r})$ depends on phase $\varphi_{DAO}(\mathbf{r})$ applied at the DWFC step and is assessed by computing an image quality (sharpness) metric J . An algorithm is used to optimize metric J , leading to an image $I_{DAO}(\mathbf{r})$ with improved quality. Details about the image formation and optimization process are presented in section 2.3.

2.2 Wavefront sensing techniques for DAO systems

In conventional AO systems the spatial resolution of the wavefront sensor output (i.e. the spacing between data points) is selected in relation to the spatial resolution of the wavefront corrector (e.g. spacing between deformable mirror actuators). Sensing of the incoming wavefront aberrations with high spatial frequency does not provide better AO performance if the corrector device is unable to match this spatial resolution. In contrast, image quality in DAO systems is directly related to the spatial resolution of the wavefront measurement. DAO systems hence require relatively high resolution wavefront sensing capabilities.

Although the resolution yielded by wavefront sensors typically used in adaptive optics such as Shack-Hartmann [3,4] or curvature sensors [5,6] does not exceed a few tens to a couple hundred data points across the system's aperture, a number of wavefront sensing techniques capable of providing high resolution outputs exist. Among them some are potentially suitable for DAO systems including: phase retrieval from sets of pupil and focal plane intensity distributions [7-9], phase diversity [10,11], schlieren techniques and phase contrast techniques [12,13] such as the Zernike filter [14-16] and the Smartt point-diffraction interferometer [17-20]. Approaches based on holographic recording of the wavefront have also been used successfully [21-23]. A sensor developed recently – referred to as multi-aperture phase reconstruction (MAPR) sensor [24] – which uses a hybrid approach between the Shack-Hartmann and Gerchberg-Saxton [7] techniques provides high-resolution measurements and is also a candidate for DAO system implementation.

A growing number of imaging applications now require operation over near-horizontal or slant atmospheric paths. These propagations scenarios are characterized by moderate to strong intensity scintillation [25-27]. This means that in addition to high resolution requirements, robustness to high scintillation levels is a key criterion for selecting sensors suitable for DAO applications. Another important criterion is the computational cost of the wavefront reconstruction algorithm as it impacts the speed of operation of the sensor. In this regard the MAPR sensor appears suitable for DAO applications since it is capable of providing high-resolution measurements under conditions of strong intensity scintillation (so called scintillation-resistant) and in the presence of branch points [28,29]. It yields an average Strehl ratio exceeding 0.9 for scintillation index values $\sigma_I^2 \leq 1.25$ and $D/r_0 \leq 8$, and 0.8 for $\sigma_I^2 \leq 1.75$ and $D/r_0 \leq 12$ and reconstruction is computationally efficient as a result of the parallel architecture of the reconstruction algorithm. Numerical results presented are obtained for complex field measurements $A(\mathbf{r})$ provided by a MAPR sensor [24].

2.3 Anisoplanatic image synthesis and compensation

As a result of anisoplanatism image quality varies significantly across the field-of-view of the system and image compensation based on phase conjugation is effective only over a small angular extent with size related to the isoplanatic angle θ_0 . An approach for performing efficient DAO compensation is to apply the technique locally over image regions that are nearly isoplanatic. We use in this section a block-by-block (mosaic) post-processing technique in which the DAO approach is applied sequentially to region (block) Ω_j and the resulting image consists in the combination of compensated image regions Ω_j into a single image corresponding to the entire FOV (region Ω). Consider an image region Ω_j defined by function

$$M_{\Omega_j}(\mathbf{r}) = \exp \left[\left(-\frac{|\mathbf{r} - \mathbf{r}_j|^2}{2\omega_{\Omega}^2} \right)^8 \right], \quad (2)$$

where $\mathbf{r}_j$ defines the position of the j^{th} image region and ω_{Ω} denoted its size. Image synthesis and DAO compensation over region Ω_j is performed based on the measurement $A(\mathbf{r})$ of the optical field in the pupil of the system and consists of the following steps:

- **Step 1: Digital wavefront correction**

A wavefront corrector phase function $\varphi_{DAO}(\mathbf{r})$ is represented as

$$\varphi_{DAO}(\mathbf{r}) = \sum_{k=1}^{N_{DAO}} a_k S_k(\mathbf{r}), \quad (3)$$

where $\{S_k(\mathbf{r})\}$ is a set of response functions for the digital wavefront corrector, $\mathbf{a} = \{a_k\}$ is the vector of commands sent to the DWFC, and N_{DAO} is the number of control channels of the DWFC. The corrector phase function $\varphi_{DAO}(\mathbf{r})$

is then applied to the complex field amplitude $A(\mathbf{r})$ provided by the CFS and results in compensated field $A_{DAO}(\mathbf{r})$ given by

$$A_{DAO}(\mathbf{r}) = A(\mathbf{r}) \exp[-j\varphi_{DAO}(\mathbf{r})]. \quad (4)$$

- **Step 2: Image synthesis**

The compensated field $A_{DAO}(\mathbf{r})$ in Eq. (4) is used for synthesis of the DAO compensated image $I_{DAO}(\mathbf{r})$. The DAO image is computed using the Fresnel approximation as follows [15]:

$$I_{DAO}(\mathbf{r}) = \left| \frac{1}{\lambda Z} \int_{-\infty}^{+\infty} A_{DAO}(\mathbf{r}') \exp \left[-j \frac{k}{2} \left(\frac{|\mathbf{r}'|^2}{F} - \frac{|\mathbf{r} - \mathbf{r}'|^2}{L_i} \right) \right] d\mathbf{r}' \right|^2, \quad (5)$$

where k is the wave number, F is the focal length of the digital lens and $L_i = (1/F - 1/L)^{-1}$ is the distance between the digital lens plane and the image plane. The term L denotes the distance between the lens and the object plane of interest.

- **Step 3: Local image quality metric computation**

The quality of the synthesized image $I_{DAO}(\mathbf{r})$ in Eq. (5) depends on the command vector $\mathbf{a}$ applied to the digital WFC [Eq. (3)]. Vector $\mathbf{a}$ can be considered as a parameter controlling the quality of image $I_{DAO}(\mathbf{r})$. Improving image quality in the region Ω_j can be achieved by optimizing a sharpness metric J_{Ω_j} given by

$$J_{\Omega_j}(\mathbf{a}) = \frac{\int I_{DAO}^2(\mathbf{r}) M_{\Omega_j}(\mathbf{r}) d\mathbf{r}}{\int I_{DAO}(\mathbf{r}) M_{\Omega_j}(\mathbf{r}) d\mathbf{r}}, \quad (6)$$

where $M_{\Omega_j}(\mathbf{r})$ is the function defining region Ω_j [see Eq. (2)]. Although the intensity-squared sharpness metric is commonly used, other criteria could be used to assess image quality such as a gradient-based metric or the Tenengrad criterion.

- **Step 4: Image quality metric optimization**

Optimization of metrics J_{Ω_j} can be achieved using various numerical techniques. We consider for example metric optimization based on the stochastic parallel gradient descent (SPGD) control algorithm [30]. In accordance with this algorithm command vector $\mathbf{a}$ update rule is given at each iteration n by the following procedure:

$$a_k^{(n+1)} = a_k^{(n)} + \gamma^{(n)} \delta J_{\Omega_j}^{(n)} \delta a_k^{(n)} \text{ for } k = 1, \dots, N_{DAO}, \quad (7)$$

where $\gamma^{(n)} > 0$ is a gain coefficient, $\delta J_{\Omega_j}^{(n)}$ is the metric response to small-amplitude random perturbations of control vector $\{\delta a_k^{(n)}\}$ applied simultaneously to all N_{DAO} DWFC control channels. The control channel updates are repeated until convergence of vector $\mathbf{a}$ toward a small vicinity of the stationary state. The number of iterations N_{it} required for convergence is defined from the common criterion

$$\epsilon(n = N_{it}) = \frac{|J_{\Omega_j}^{(n)} - J_{\Omega_j}^{(n-1)}|}{J_{\Omega_j}^{(n)}} \leq \epsilon_0 \ll 1, \quad (8)$$

and the resulting control vector for compensation of region Ω_j , denoted $\mathbf{a}_{\Omega_j}$, corresponds to the vector obtained at the last iteration: $\mathbf{a}_{\Omega_j} = \mathbf{a}^{(n=N_{it})}$. Optimization over the entire image region Ω can be achieved by repeating sequentially steps 1 through 4 for each region Ω_j and results in a set of control vectors $\{\mathbf{a}_{\Omega_j}\}$.

In an ideal compensation scenario the DWFC phase $\varphi_{DAO}(\mathbf{r})$ resulting from the optimization process would compensate the turbulence-induced phase aberration so that $\varphi_{DAO}(\mathbf{r}) = \varphi_{turb}(\mathbf{r})$ [see Eq. (1)]. In this ideal case only phase

information related to the object being imaged remains in the compensated field: $\arg[A_{DAO}(\mathbf{r})] = \varphi_{obj}(\mathbf{r})$ and leads to optimal image quality.

3. PERFORMANCE ANALYSIS

In this section performance of DAO systems is analyzed using a numerical simulation and results for various turbulence strengths are discussed.

3.1 Numerical model

Performance was evaluated from an ensemble of digitally-generated random complex fields used as input optical waves to the DAO system. For each realization of the input field the complex amplitude in the DAO system pupil plane $A_{in}(\mathbf{r})$ (see Fig. 1) was obtained using the conventional split-operator approach for simulating wave optics propagation through a volume of atmospheric turbulence [31]. At the beginning of the propagation path ($z = 0$) we used a monochromatic optical field with complex amplitude $A_{prop}(\mathbf{r}, z = 0) = I_{obj}^{1/2}(\mathbf{r}) \exp[j\varphi_{surf}(\mathbf{r})]$, where $I_{obj}(\mathbf{r})$ is the intensity distribution of the object being imaged and $\varphi_{surf}(\mathbf{r})$ is a random phase function uniformly distributed in $[-\pi, \pi]$ and δ -correlated in space. The term $\varphi_{surf}(\mathbf{r})$ is used to model the “optically rough” surface of the object. The optical field complex amplitude at the end of the propagation path ($z = L$) was utilized as the DAO system input field: $A_{in}(\mathbf{r}) = A_{prop}(\mathbf{r}, z = L)$. Optical inhomogeneities along the propagation path were modeled with a set of 10 random phase screens corresponding to the Kolmogorov turbulence power spectrum. We considered a horizontal propagation scenario so the phase screens were equally spaced along the propagation path and their impact (i.e. turbulence strength) was characterized by a constant ratio D/r_0 where D is the diameter of the DAO system aperture and r_0 is the characteristic Fried parameter for plane waves. By modifying ratio D/r_0 one can control the strength of input field phase aberrations. In the numerical simulations, D/r_0 ranged from zero (i.e. free-space propagation) to 10. Figures 2(a)-2(d) show examples of the input field intensity and phase distributions that are obtained using the technique described above for $D/r_0 = 4$ [(a) and (b)] and for $D/r_0 = 8$ [(c) and (d)]. Note that the phase distributions in Figs. 2(b) and 2(d) contain phase discontinuities (branch points). Images were obtained for a propagation distance $L = 0.05L_{diff}$ where $L_{diff} = k(D/2)^2$ is the diffractive distance, $k = 2\pi/\lambda$ is the wave number, and λ is the imaging wavelength.

In the DAO system, digital wavefront compensation function $\varphi_{DAO}(\mathbf{r})$ is computed as a weighted sum of response functions $S_k(\mathbf{r})$ [see Eq. (3)]. In our numerical simulation these response functions are taken as scaled Zernike polynomials: $S_k(\mathbf{r}) = Z_k(Q\mathbf{r})$, where $Z_k(\mathbf{r})$ is the Zernike polynomial with index k and Q is a scaling factor. Since Zernike polynomials are defined onto the unit circle the term Q is chosen as $Q = D/2$ so that response functions $S_k(\mathbf{r})$ are defined over the aperture area of the DAO system. In the numerical analysis we used $N_{DAO} = 36$ Zernike polynomials corresponding to the first 8 Zernike modes. Note that the first Zernike polynomial (piston) does not impact image quality and does not need to be included in the construction of $\varphi_{DAO}(\mathbf{r})$. Similarly, Zernike polynomials with index $k = 2$ and $k = 3$ (tip and tilt) do not have an influence on image quality or its metric. However, tip-tilt wavefront aberrations in the pupil plane cause a global shift of the intensity distribution in the image plane. In order to compensate for this, image alignment is performed in our numerical simulations by mean of a conventional registration technique [32] using an ensemble average image as a reference.

The quality of the DAO-compensated image $I_{DAO}(\mathbf{r})$ is assessed using a sharpness metric defined as

$$J = \frac{\int I_{DAO}^2(\mathbf{r}) d\mathbf{r}}{\int I_{dl}^2(\mathbf{r}) d\mathbf{r}}, \quad (9)$$

where $I_{dl}(\mathbf{r})$ is the image that would be obtained in the absence of atmospheric turbulence (i.e. diffraction-limited image). In the case of an ideal compensation the turbulence-induced wavefront aberrations are fully corrected by the DWFC [$\varphi_{DAO}(\mathbf{r}) = \varphi_{turb}(\mathbf{r})$] and $J = 1$.

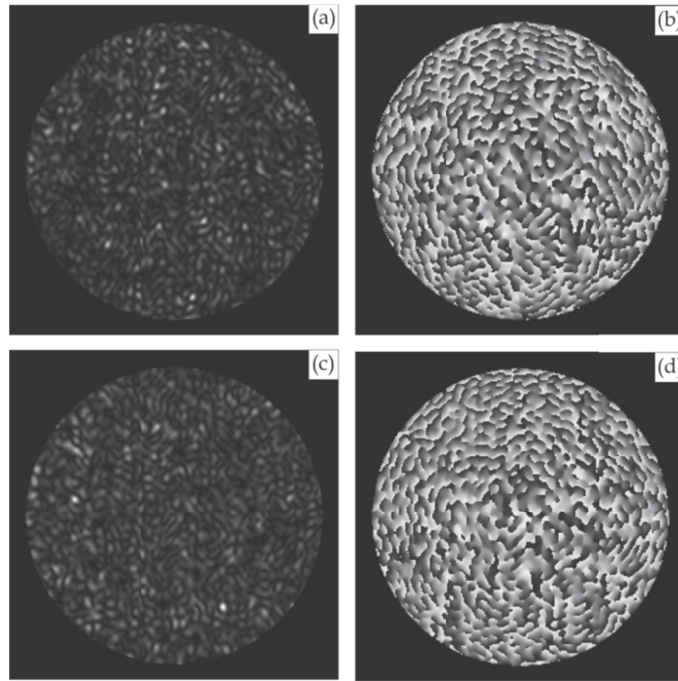


Figure 2. Intensity (left column) and phase (right column) distributions of the computer-generated complex field $A_{in}(\mathbf{r})$ used as input of the DAO system for $D/r_0 = 4$ [(a),(b)] and $D/r_0 = 8$ [(c),(d)]. Phase distributions are shown in a 2π range [between $-\pi$ (black) and $+\pi$ (white)].

3.2 DAO system performance

In this section we analyze results of image synthesis and compensation using the DAO approach described in section 2 using the numerical model presented in section 3.1. In order to illustrate the effect of anisoplanatism we first consider DAO performance over an image region Ω_j that is nearly isoplanatic in size. Later we consider processing of the entire image region Ω (anisoplanatic conditions). Figure 3 shows intensity distribution $I_{DAO}(\mathbf{r})$ and cross-sections for an object with intensity distribution consisting of point sources arranged in a 7-by-7 array. The resulting intensity distributions hence correspond to the average DAO-compensated PSF of the system for different values of the field angle θ , and is showed for $D/r_0 = 4$ [Figs. 3(a) and 3(b)] and for $D/r_0 = 8$ [Figs. 3(c) and 3(d)]. As a result of anisoplanatism the PSF is space-varying and its distribution broadens significantly as the angular separation θ with the metric region Ω_j increases. Also, the degradation occurs more rapidly with respect to angle θ as turbulence strength increases [compare cross-sections in Figs. 3(b) and 3(d)]. Results were obtained by averaging the resulting images $I_{DAO}(\mathbf{r})$ for 100 realizations of the random phase screens.

As described in section 2.3 compensation is based on the optimization of metric J_j using an iterative SPGD algorithm. Figure 4 shows average convergence curves for metric J_j as a function of the number of SPGD iterations performed. The two curves displayed correspond to the optimization of image regions Ω_j shown in Figure 3 for $D/r_0 = 4$ and $D/r_0 = 8$. In both cases the metric value approximately doubled during the optimization process.

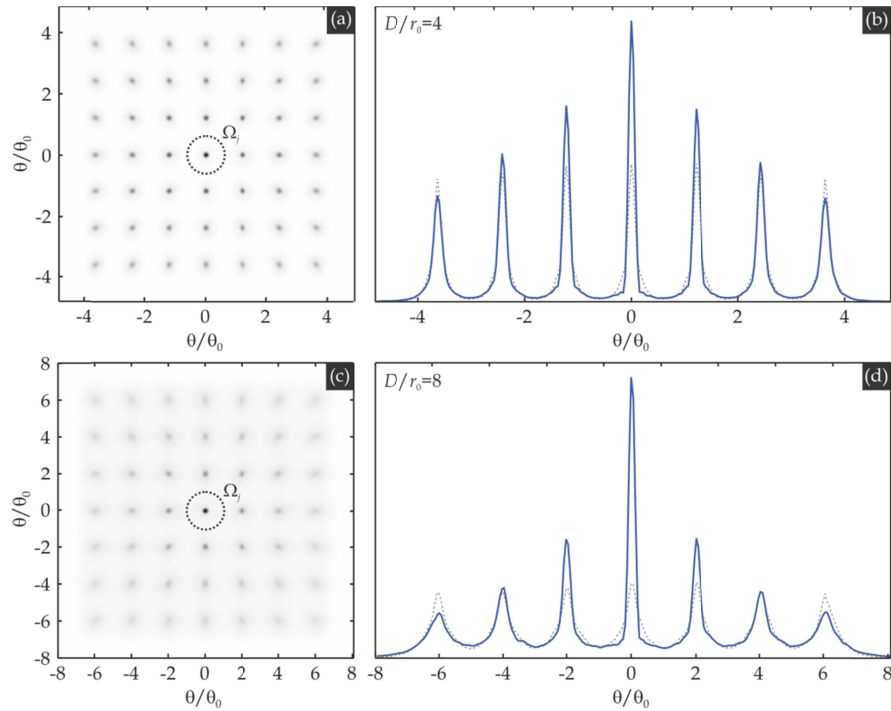


Figure 3. Average DAO-compensated intensity distributions (left column) for an array of 7-by-7 point sources for $D/r_0 = 4$ (top row) and $D/r_0 = 8$ (bottom row). The column on the right depicts the average uncompensated (dotted curve) and DAO-compensated (solid curve) cross-sections of the PSF's through the center of region Ω_j . Image compensation is based on optimizing image quality metric J_j defined over region Ω_j . Region Ω_j has a diameter ω_Ω of approximately $1.2\theta_0$ and $2\theta_0$ for $D/r_0 = 4$ and $D/r_0 = 8$ respectively (see circled areas).

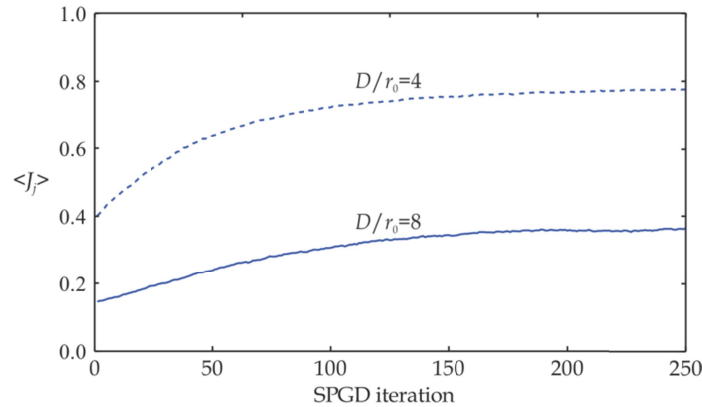


Figure 4. Average convergence curves for metric J_j . The curves displayed correspond to optimization of image region Ω_j shown in Fig. 3 for $D/r_0 = 4$ and $D/r_0 = 8$.

We consider now the performance of DAO systems under anisoplanatic conditions. Figures 5 and 6 display images prior and after DAO compensation for the point source object used in Figure 3 and for an USAF resolution chart respectively. Results are shown for two turbulence conditions: $D/r_0 = 4$ and $D/r_0 = 8$ which correspond to a full field-of-view of approximately $10\theta_0$ and $16\theta_0$. In both Figures significant image quality improvements can be observed with an increase of the metric J of 70% (point source object) and 49% (USAF object) for $D/r_0 = 4$, and 123% and 33% for $D/r_0 = 8$.

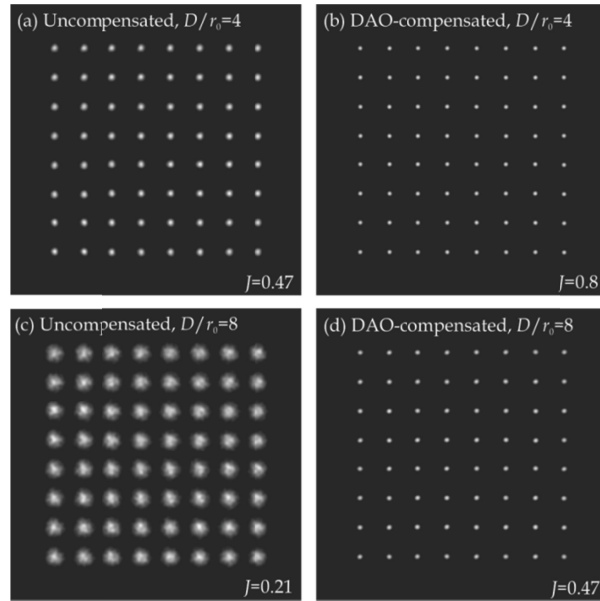


Figure 5. Image of an array of point sources prior and after anisoplanatic DAO compensation for $D/r_0 = 4$ (top row) and $D/r_0 = 8$ (bottom row).

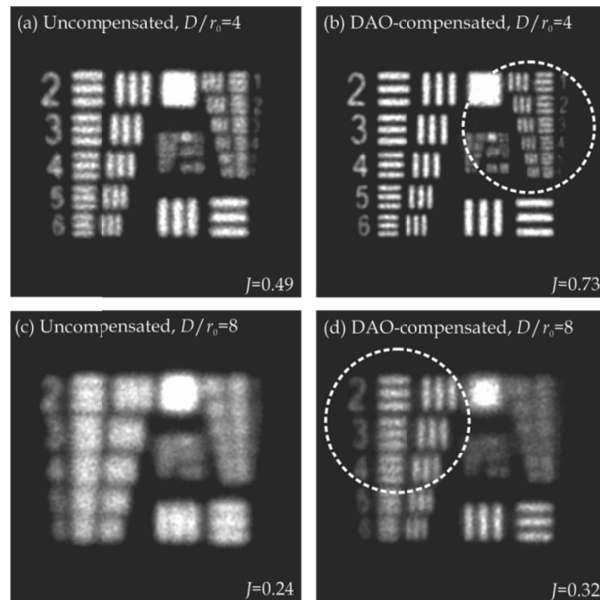


Figure 6. Image of an USAF resolution chart prior and after anisoplanatic DAO compensation for $D/r_0 = 4$ and $D/r_0 = 8$. Note that the DAO compensation process reveals image details as pointed out by the circled areas.

4. CONCLUSION

We presented an alternative technique to conventional adaptive optics imaging schemes which we refer to as digital adaptive optics. The technique consists in a two-step process. First, an optical sensor provides a measurement of the wave's complex amplitude (intensity and phase distributions) in the pupil of the imaging system. Second, a digital post-processing algorithm is applied to the complex field measurement in order to synthesize an image and mitigate the effect of atmospheric turbulence. This final step is based on the optimization of an image quality metric and compensation of the wavefront aberrations is performed in a numerical manner. While the conventional AO approach compensates

aberrations in real-time the DAO operates as a post-processing scheme. DAO systems has the advantage of requiring simpler and less costly implementations since they do not require opto-mechanical wavefront correctors and their real-time control hardware, but this also means they are primarily suited for applications that do not require real-time operation.

Performance of DAO systems was evaluated by mean of a numerical analysis. The analysis revealed the DAO approach can significantly improve image quality even in strong turbulence conditions. The block-by-block processing technique presented was shown to be effective for image synthesis and compensation under anisoplanatic scenarios.

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